The Mathematics Education into the 21st Century Project

Proceedings of the International Conference

The Humanistic Renaissance In Mathematics Education

Editor
Alan Rogerson

September 19-25, 2003
Hotel Continental, Brno, Czech Republic

PRACTICAL APPLICATION OF THE ETHICS IN GEOMETRY
.....pp. 105-109

PRACTICAL APPLICATION OF THE ETHICS IN GEOMETRY

Bradford Hansen-Smith
4606 N. Elston #3
Chicago IL 60630, USA
E-mail: brad@wholemovement.com

The concept of Wholeness has not been seriously considered as an intuitive and legitimate place from which to start formal education. This concept of the Whole is not about the endless number of wholes we talk about as coherent systems, defined by various set theories, and codified concepts of relative importance; rather it is inclusive of the all-we-know-and-even-more-than-we-will-never-know Whole. It is about the "biggest picture", as many educators like to say. We need to understand that the biggest picture possible of which anyone can conceive must be origin to the single endless movement we call creation. The Wholeness of infinite intelligence, that is magnitudes beyond human imagination, is personalized to each of us in the concept of God. We have individually been created as personal beings. Our nature is to relate personally, so our relationship to creation and creator must be of a personal nature, it cannot be otherwise.

Our relationship to the universe is partially influenced by our understanding about mathematics, as much for what it proposes as for what it leaves out. This is not an issue of religious doctrine; it is about the on-going human need for personal understanding of our own existence. This biggest picture needs to be integrated into curriculum and talked about in the classroom. Folding circles is the most practical and experiential approach to understanding the concept of Wholeness. By beginning this exploration at the first grade level, we can easily introduce geometry, mathematics, and the ethics of human interaction. This paper addresses how this can be done. Mathematics is an activity of the mind looking for truth through rules of its own devising. However, that activity has become separated from body, out of touch with spirit, and without ethical reference. A synthesis is possible through a comprehensive approach to developing the mind.

In re-evaluating mathematics education we have the opportunity to help reshape how we think about the universe, and how interaction might occur differently. The abstract concepts of higher mathematics are represented in the words we use. Words develop from concrete experience, and without some agreement about core experience, the abstract words mean little. Spatial geometry is aligned to patterns we experience in the world; Through greater familiarity with these patterns we can expand our understanding. Wholemovement is a word that comprehensively describes geometry.[i] This self-referencing movement of the Whole is inclusive, it is the biggest picture. Folding the circle reveals a process that models this concept of Wholemovement.

THE CIRCLE

The only shape inherently Whole is the reformation of a sphere compressed to a circle disk in space. We approach the circle in space as a part. It is also Whole, somewhat similar to light we observe as both particle and wave. The circle is both a part of and is the impersonal Whole. To draw the image of a circle is a static activity which generates nothing. Cut the image from the paper and it becomes dynamic, a self-referencing boundary of compressed spherical information. As we fold the circle to itself, a familiar world of information is revealed in surprising ways.

The circle functions as both Whole and part in a unique way. 2-D construction is revealed in the folded lines; 3-D transformation is the reconfiguring of the folded lines. The patterns and forms, inherent to the folds of the circle, model the fundamental geometry of how we identify, in all the ways we understand, the physical universe. Folding the circle in half reveals qualities that are principal,[ii] because they happen first. From this principled first fold, all configurations of the circle are generated. These principles are fundamental to geometry, they are the context for understanding mathematical concepts, and can be demonstrated by sequentially folding the circle. To regard the circle as Whole yields far greater potential than calling it zero; fullness offers more benefit than having it empty. The greater meaning of mathematics is a reflection of the biggest picture. Anything less than the Whole is limited to describing parts.

There is no disorder in folding the circle. It reflects a hierarchy of pattern formation which demonstrates order and intelligence of a nature far greater than the human mind. The unity of the circle reveals parts which form a patterned environment for endlessly generating more parts. In the same way we are continually reforming the environment for each other, by the choices we make. The unity of relationships is not in the parts. Unit is found within the Whole and is directive towards appropriate interrelationships between parts. Through physical, mental, and spiritual attainment we strive to achieve a larger understanding of the origin of our universe. This requires a faith in what is greater than ourselves. Agreement on what is comprehensively Whole will bring about a greater acceptance of differences, from which we will find higher value. It is in the comprehensive nature of the folded circle that we find demonstration of the principles that give meaning to the values that we seek.

ETHICS IN GEOMETRY

Ethics is a *universal structural imperative*. It is the quality of conscious interaction that supports a constantly changing universe. All movement is generated by, and supported through, principled patterns of intelligent design. Interaction contrary to established design limits growth. Alignment with universal movement comes through understanding.

Centuries of developing extreme religious positioning have left us confused as to the understanding of God as the personalization of the biggest and greatest idea we can have about who we are, where we are, why we are, and even how it is that we are at all. Such positioning has led to misguided notions of education, to teaching opinions and partial observations as predetermined facts, rather than teaching principles of origin. Ethics is the "biggest picture" and will always move towards the greatest benefit to the greatest number of parts (including ourselves), over the longest period of time.

We teach from a reduced context where everything is relative. It is crucial to teach our children from the most comprehensive place possible, to give their minds the greatest stretch, and provide them with principled activities that reflect intelligence greater than our own. Understanding of the Wholemovement of the circle will enlarge the meaning of mathematics. Potential is lost when we do not see the Whole, when we avoid being involved with the greatest ideas we can have. The activity of folding circles requires us to see differently. We move towards deeper levels in observing, reflecting and modeling. The physical act of folding the circle shows inherent *beauty* and intelligence design that reflects an absolute *truth* expressed in the *goodness* of creative action. Folding the circle reveals principles and a process beyond human construction, essential to developing our individual and collective potential, reaching beyond what is immediate.

With each fold and every reformation of the circle, we engage in movement towards realizing the endless potential of formation inherent within the circle. This interaction supports the ongoing expression of the circle, and is supported by the Wholeness of the circle. The multiple functions of all discernable parts, as they form various combinations of individual systems, are all interdependent and ordered to spherically-compressed patterns. Folding and joining multiple circles extends potential outward. Folding one circle to a higher frequency grid extends potential inward. The fulfillment of all into/out-from individualized potential is through the interconnectedness; the touching of parts in alignment with the movement of the Whole.

When we impose our will against the creased grid in the circle, movement jams up, patterns cease to generate. It is important to feel the movement of the circle through creasing and reforming. It is also important to feel the movement of numbers and words, to feel the movements of our lives. When we attempt to impose our will, things jam up and movement ceases. It is not possible to move one part of the circle without relationships shifting and changing. An aspect of ethical choice is to be aware of the direct relationships which have gone before, and all that is yet to come, and how it all interacts with every part effecting all others. Ethics is about the choice of direction and appropriate interaction in the development and support of parts within ever expanding larger parts. This is a fractalized, reciprocal balance where all level of parts are sustained by the Whole. Only through the individualization of division can this happen.

Absolute principles need to be understood and agreed upon as contextual reference, for the same reason we agree on the symbols used to express specific concepts in mathematics. Without agreement on this basic level, there is no sustainable community. The circle is unity to all formation. As parts of the circle are folded out of sight to show various forming of polyhedra, the circle itself always remains Whole. It is through continuous movement of the Whole that potential can be given expression.

The more we understand about the Whole, the greater will be our understanding of parts. With greater awareness of pattern comes greater potential for diversity and interrelated formations of systems, and understanding of the abstraction of information. When we start with parts we end with parts. We have to start with the Whole to understand Wholeness. This is a fundamental educational choice we will eventually have to make. Students drawing pictures of circles is not enough; they need to cut them out and fold them, observe them, talk about them, and find value in the meaning.

Principles, as comprehensively as we can understand, will always precede the facts of individual experience. Without direct experience, principles mean little. We find much value in the extraordinary products of mathematics, the arts, sciences, athletic performance, and in all areas of human achievements. All this effort towards collective potential is revealed through individual choices of interaction. Truth, beauty, and goodness in the choices we make, are qualities that go beyond human invention. They reflect the depth of our understanding, add meaning to what we do. These qualities need to be incorporated into mathematics education. We need to reconsider the value of these qualities in our understanding of the universe, to make the broader connections to mathematical thinking that will give greater benefit to our lives and expand mathematics in ways that will allow deeper insights.

IN THE CLASSROOM

Ethics is inherent in geometry. We need to discuss ethics in the classroom so students know we consider it to be important. Five and six year old children can talk about spheres and circles and what is Whole. They have ideas about truth, beauty and what is good. They need to observe these qualities in nature, to see them reflected in geometry and to see how they are demonstrated in mathematical expressions. Play with questions: Why do equations take the form of two different things being equal? In what ways do we say two people are equal? What is equally balanced between things that are different? What is unequal about two things that look the same? Where is the truth, what is the beauty, and to what good is AB=BA?

Children need be told the reason they are learning these things is not because these things are important to know, rather it is because the children themselves are important. These things they learn are gifts to enrich and deepen their lives. The circle is a gift because we do not have to know anything to get so much information from it. Everything is in this compressed ball to be discovered and played with, if we so chose.

Questioning assumptions, axioms and rules as they come up makes mathematics more interesting, understandable, and relevant. The more connections students can make, the greater will be the meaning from which they gain value. Through folding circles, students discover mathematics for themselves: one fold, one question, and one unexpected connection at a time, all within the greatest context possible.

Four balls can be used to demonstrate cell division, a singular movement without separation reforming into four spheres. This shows the tetrahedron pattern and the beginning of geometric progression, simple arithmetic. Four spherical locations and six points of connections equal the number ten. The first diameter fold in the circle is one and zero. Write the numbers one through ten on the board. First grade students can tell you what is inconsistent, what is not right with the pattern of numbers. Get rid of the one in the ten and put the zero in front; ten places remain, consistently all single digits. If we talk about the circle as Whole they get it more quickly, knowing that the circle comes first before the numbers. They see a consistent order in the sequence of 0-9.

East Indian mathematician Ramanujan said, "Zero it seemed, represented Absolute Reality. Infinity, or ∞, was the myriad manifestation of that Reality. Their mathematical product, ∞ x o, was not one number, but all numbers, each which corresponds to individual acts of creation."[iii] In their own way these young students express similar understanding. Zero is a circle representing Absolute Reality. There must first be Wholeness before division can occur. Through division there are multiple parts that can be added and subtracted. Having folded the circle in half, students count parts finding *one* within the circle, then discovering *two*, *three*, and *four*. They also find a tetrahedral pattern of *ten* that reflects the number of locations in the four spheres.

Students understand the circle is Whole by talking about what they see as they compare it to other shapes. When they touch two points on the circumference together and crease the circle, they talk about what has occurred, what has been generated that was not there before folding. The straight-line happens inside the circle. The circle and line are not separated as we see in the symbol for ten. Within the circle they count 1 line, 2 parts that are in the form of triangulation, 3. A ratio of one whole to two parts is formed. Two points touched together generate two more points making 4 points in space, a tetrahedron pattern without separation. (1+2+3+4=10) There are eight triangles flat, four

triangles spatially. The creased line is a relationship half way between at right angle to the movement of two points touching. We discuss all this in the students' own words. Movement at minimum goes in two directions. This means that what you touch will at the same time, touch you. The diameter is also an axis forming a spherical pattern. The first fold goes in both directions and is a curved movement in space showing the origin or the circle, the sphere, and where it is going by forming a tetrahedron pattern. By counting the four points, the five unformed connecting lines, plus one fold, we get the number ten. The first fold of the circle is rich in information and is accessible when we talk about what we see, connect to what we know, look for what we do not know, knowing it is all there.

The infinite nature of the circle is the concentric movement into and out from itself. My first grade students figured out that it is like putting the circle first and all the numbers will follow, only in both directions. If there are endless numbers of circles going out and endless numbers of circles going in, the circle has no center, no boundary. The circle we start with is our point of reference circle. In that way we will never be lost. We will know that we are always within the Wholeness of the circle.

The Fibonacci series, like 1-10, is inconsistent; there is no causal reference. It usually begins with two ones, like rubbing sticks together; 1, 1, 2, 3, 5.... Start with the circle, one fold generates duality in triangulation; O, 1, 2, 3. It is like a game of catch-up; O, 1, 2, 3, 5, 8, 13... Everything is absolutely consistent and self-referencing as the numbers accumulate what has been towards what will become. This is an accurate representation of growth with a defined origin. The 1, 1, 2, 3, 5, 8... is the form of the interval spaces between the numbers; it is without origin. First graders will understand this, having discovered O-9. They are just starting to learn numbers and a part of that is the fun in playing with them, exploring different kinds of series, what they do and don't do. It is much easier to blend different concepts using the same skills at the primary level than to teach separately years later, when students are struggling to make connections.

The word diameter means to divide into two equal parts, which is what folding the circle in half does. We can rename the line by the two end points, A and B. Observe and discuss line AB. It is the same as line BA. Students see the difference is direction of movement. It is an easy jump to AB=BA. There are three ways to get from point A to point B and three ways back again; six distinct movements from one folded line. This gets us into curved paths and areas where each can have its own code name, like area 1 and area 2. We make simple equations like $\overline{AB} + \overset{\frown}{BA} = a1$, or ½ O. First grade students have fun making up this code language. It allows them to find more relationships between parts than they have first observed using words. After folding three diameters into the circle we can give code names to angles as well as the points, lines and areas. The relationships become more complex as they explore their new language. The only rule is that everyone must agree on what the code is and agree when symbols are added or changed. This is a social activity. It requires close observation, seeing value in differences, finding interesting relationships, and then communicating to each other what they find with their new language.

With the three folds of the hexagon pattern we again count the main parts. There are three diameters, (draw a 3 on the board). They make six divisions (draw a six next to the three, 36). It happens in the circle, (draw a circle next to the six, 36O). Now we have the numbers that represent the individual parts. (I put a little circle to the upper right of the 36O°). This is to remind us that whenever we see this number with the little circle it

means the parts of the circle. With the numbers to represent the circle and the six major areas of division, we can easily figure out how many parts of 360° each area has.

The first fold of the circle with the right angle movement establishes the xy axis. The folded circle becomes a graph. The inside/outside reciprocal function of the first fold demonstrates positive and negative positions, to the right and left of the diameter, giving demonstration to right hand and left hand symmetry of right triangles. From the folding of the tetrahedron we discover the Pythagorean theorem as we talk about the different kinds and functions of triangles. We don't call it a theorem; it is just what right triangles do in context with everything else. The first fold is a right angle movement that generates right-angle triangles. Students begin to see a difference between pattern and form.

It is easy to make connections in the circle; every fold generates new information as a continuous individualization of parts without separation, all in the same place. Discussion and observations might come after folding and joining circles, or step by step along with each fold. The code language can plug in anywhere when appropriate. The circle is fun to color as another way to code and keep track of relationships while exploring the design possibilities in the folded grid. The comprehensive nature of the circle makes it easy to interject folding anywhere in the curriculum. Observation and discussion is always necessary, otherwise information remains hidden. Using the circle, students invent and develop mathematics for themselves. In this way the teacher guides the students towards self-learning. When a child can fold a circle in half they can talk about what they see, make connections, and discover new things for themselves.

A MATHEMATICAL IMPERATIVE

The reality of the Wholeness of life needs to be incorporated into the understanding and teaching of mathematics. The circle is where we start. The concept of the Absolute is beyond human understanding, but it is inherent in the circle. It can offer direction and guidance towards appropriate action in the choices we have to make. As chords are individually folded into the circle and proportionally rearranged in diverse combinations, they function harmoniously in ways that support and inform the circle. Each individual part is dependent upon the circle, which is expressed in endlessly developing points and lines, that by themselves do not exist.

Principles order the patterns which form spatial arrangements in the circle, that gives demonstration to the ethical nature of interaction. How we view our personal relationship to the Whole determines the choices we make. Our greatest thoughts give context and life to all the small thoughts we have about the physical, mental, and the spiritual nature of our existence. Truth, beauty, and goodness are the inherent qualities of patterns in mathematics about which we seek to understand and to give expression. Once we begin to find ways to meaningfully incorporate the ethics in geometry into mathematics education, our minds, and the minds of our students will open to a greater universe yet to be discovered.

FOOTNOTES

---

[i] Bradford Hansen-Smith,(2002) *Wholemovement of the circle*, The Mathematics Education into the 21st Century Project; Proceedings of the International Conference, The Humanistic Renaissance in Mathematics Education, editor Alan Rogerson. Sept. 2002, 171-175

[ii] Bradford Hansen-Smith, (1999) *The Geometry of Wholemovement; folding the circle for information*, Wholemovement Publ. 17

[iii] Robert Kanigel, 1991, *The Man Who Knew Infinity, A life of the genius Ramanujan* Washington Square Press.